U.S.S. Nagurskiy (2)

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COULOMB RELAXATION OF ELECTRONS OF EARTH'S RADIATION BELTS IN THE DENSE LAYERS OF THE ATMOSPHERE

by

G. A. Timofeyev

(USSR)

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SUMMARY

The temporal dependence of electron distribution, conditioned by the Coulomb interactions, is studied with the aid of the kinetic equation. The lifetime of electrons with energy 1 Mev is obtained for various lines of force of the geomagnetic field at reflection altitudes of 350, 500 and 750 km.

* * *

The Coulomb interaction of charged particles with atmosphere matter is one of the essential processes leading to particle loss in Earth's radiation belts. As a result of Coulomb interactions the fast charged particles slow down, whereupon by virtue of atmosphere inhomogeneity, the particles reaching during their motion through the geomagnetic field lower altitudes above ground decelerate more rapidly. This is why the scattering, leading to depth variation of particle penetration into the atmosphere, play an important role.

A series of works are devoted to the question of the Coulomb interaction of radiation belts' electrons. The lifetime of electrons in Earth's radiation belts is obtained in the assumption that, as a result of the scattering of these electrons near the reflection point, the latter's drift into denser layers of the atmosphere takes place [1]. The relaxation in the distribution of fast electrons with reflection points at great altitudes is studied in ref. [2 - 4]. The question of time dependence of the distribution function of electrons with reflection points at low altitudes is considered in ref.[5]. It is shown that the scattering of electrons of radiation belts should be considered as a diffusion process. The Coulomb interactions of electrons in dense layers of the atmosphere are also considered in [6]. The works [7, 8] are devoted to the study of Coulomb interactions in artificial radiation belts.

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* (KULONOVSKAYA RELAKSATSIYA ELEKTRONOV RADIATSIONNYKH POYASOV ZEMLI V PLOTNYKH SLOYAKH ATMOSFERY)

Studied in the present work is the time dependence of the distribution function of Earth radiation belts' fast electrons that are reflected at the initial moments of time at altitudes < 1000 km. It was assumed that, in the absence of collisions during the motion of particles in radiation belts, the adiabatic motion invariants are preserved.

For initial equation, we have the kinetic equation, in which the collision integral is written in Landau form [9]. For the collision integral of a relativistic electron with the matter, the following expression is obtained in a spherical system of coordinates:

$$S = 1/p^2 \frac{\partial}{\partial p}\{p^2 F(\mathbf{r}, p) f(\mathbf{r}, p, \vartheta, t)\} + \frac{\nu(\mathbf{r}, p)}{2\sin\vartheta}\frac{\partial}{\partial\vartheta}\left\{\sin\vartheta \frac{\partial f(\mathbf{r}, p, \vartheta, t)}{\partial\vartheta}\right\},$$

$$F(\mathbf{r}, p) = \frac{4\pi e^4}{mv^2}\left[\sum_i Z_i n_i L_i + n_e L_{\text{п.л}}\right],$$

$$\nu(\mathbf{r}, p) = \frac{4\pi e^4}{p^2 v}\left[\sum_i (Z_i^2 + Z_i) n_i L_i + (n_e + n_{ion}) L_{\text{п.л}}\right].$$

Here $f(\mathbf{r}, p, \vartheta, t)$ is the distribution function of electrons; p is the relativistic pulse; ϑ is the polar angle; F(r, p) is the force of friction of electrons in the medium; ν(r, p) is the collision frequency of electrons in the medium; v is the velocity of the electron; e and Ze are the charges of the electrons and of a particle, with which the electron interacts; L_i and $L_{\text{п.л}}$ are respectively the Coulomb logarithm for neutrals and the plasma; n_i, n_e, n_{ion} are the concentrations of neutrals, electrons and ions. The time between the consecutive collisions of radiation belts' electrons with the atmosphere matter is much greater than the oscillation period between the reflection points. This is why the integral of collisions may be averaged by the period of oscillations [10]. Upon averaging, the kinetic equation, describing the Coulomb interaction of radiation belts' electrons in the atmosphere, is written in the form

$$\frac{\partial f(p, z, t)}{\partial t} = \frac{1}{p^2}\frac{\partial}{\partial p}\{p^2 F(p, z) f(p, z, t)\} + \frac{1}{2z}\frac{\partial}{\partial z}\left\{zK(p, z)\frac{\partial f(p, z, t)}{\partial z}\right\}, \quad (1)$$

where $z = \arcsin\vartheta_0$; ϑ_0 is the pitch-angle on the equator; f(p, z, t) is the distribution function of electrons in the pulse space, normalized to the element in the phase volume $p^2 \sin\vartheta_0 dp d\vartheta_0 dV$; F(p, z) and K(p, z) are respectively the electrons' force of friction and the diffusion coefficient in the pulse space, averaged by the oscillation period of the electron. An equation analogous to (1) was also obtained in ref.[2,3]. The values of F(p, z) and K(p, z) are given by expressions

$$F(p, z, R_e) = \frac{4\pi e^4}{mv^2}\frac{1}{T(p, z, R_e)v}\times$$
$$\times\left\{\oint\left[\sum Z_i L_i n_i + n_e L_{\text{п.л}}\right]\frac{1}{\sqrt{1 - z^2 b(l)}}dl\right\}, \quad (2)$$

$$K(p,z,R_e)=\frac{4\pi e^4}{p^2 v}\frac{1}{T(p,z,R_e)\,v}\times$$

$$\times\left\{\oint\left[\sum_i (Z_i^2+Z_i)L_i n_i+(n_e+n_{ion})L_{\text{п.э}}\right]\frac{\sqrt{1-z^2 b(l)}}{b(l)}\,dl\right\},$$

$$T(p,z,R_e)=\frac{1}{v}\oint\frac{1}{\sqrt{1-z^2 b(l)}}\,dl,$$

where $T(p, z, R_e)$ is period of oscillations of the electron; l is its coordinate counted along the line of force from the equator; $b(l)$ is the ratio of the field at the point l to the field at the equator. Integration is performed over the electron path along the line of force for the oscillation period. Function $f(p, z, t)$ satisfies the boundary conditions

$$f(p,z_{min},t)=0, \tag{5}$$

$$f(p,z_{max},t)=0. \tag{6}$$

Boundary condition (5) corresponds to the fact, that because of fast diffusion of electrons into the dense atmosphere layers, at low latitudes, beginning from a certain altitude h_{min}, the number of electrons is negligibly small. Boundary condition (6) is superimposed on the basis of the fact that electrons with reflection points at low altitudes lack the time to diffuse to greater heights during the time of distribution relaxation.

A method has been worked out for the numerical calculation of Eq,(1). We passed in Eq.(1) to the function $u=\gamma^2 f(p,z,t)$, where $\gamma^2=(1-v^2/c^2)^{-1/2}$. Function u satisfies the equation of the form

$$T(x,y,t)\frac{\partial u(x,y,t)}{\partial t}=F(x,y,t)\frac{\partial u(x,y,t)}{\partial x}+$$
$$+\frac{1}{y}\frac{\partial}{\partial y}\left\{K(x,y,t)\frac{\partial u(x,y,t)}{\partial y}\right\},$$

which was represented by a system of finite-difference equations [11]

$$T_{ik}^{j+1}\frac{u_{ik+1/2}^{j+1}-u_{ik+1/2}^{j}}{\Delta t_j}=F_{ik+1/2}^{j+1}\frac{u_{ik+1}^{j+1}-u_{ik}^{j+1}}{\Delta x_k}+$$
$$+\frac{1}{2y_i}\{\Delta(K\nabla u)_{i,k+1}^{j+1}+\Delta(K\nabla u)_{ik}^{j+1}\},$$

where

$$\varphi_{ik}{}^j=\varphi(x_i,y_k,t_j),\ \varphi_{ik+1/2}^{j}=1/2(\varphi_{ik+1}^{j}+\varphi_{ik}{}^j),$$

$$\Delta t_j=t_{j+1}-t_j,\qquad \Delta x_k=x_{k+1}-x_k,\qquad \Delta y_i=y_{i+1}-y_i,$$

$$\Delta(K\nabla u)_{ik}{}^j=\frac{1}{\overline{\Delta y_i}}[A_{i+1,k}^{j}u_{i+1,k}^{j}-(A_{i+1,k}^{j}+A_{ik}{}^j)u_{ik}{}^j+A_{ik}{}^j u_{i-1,k}^{j}],$$

$$\overline{\Delta y_i}=1/2(y_{i+1}+y_{i-1}),\qquad A_{ik}{}^j=\frac{1}{\Delta y_i}K_{ik}{}^j.$$

The solution of the system (7) was found by electronic computer. When computing, the initial distribution by pitch-angles and the energy spectrum may vary within a broad range.

We chose for the initial distribution a function with sharp maximum by energy, as well as by pitch-angles. The altitude h_0 of the reflection point in the initial distribution maximum was assumed for the various calculation variants to be equal to 350, 500 and 750 km, and the energy E_0 in the distribution maximum was assumed to be 1 Mev. The distribution half-widths by reflection point heights constituted about 50 km for electrons with reflection points at 350 and 500 km and about 100 km for electrons with reflection points at 750 km altitude. The half-width of energy distribution was about 250 kev. The solution obtained as a result of the calculation may be considered approximately as the source function of Eq.(1). The source function found describes the behavior in time of the density of electrons of given initial energy with given initial pitch-angle and reflection point altitude values. The initial distribution and the solution of Eq.(1) satisfy boundary conditions (5) and (6). The value $z = z_m$ in boundary condition (5) was chosen corresponding to reflection altitude h_{min} = 200 km.

The collision frequency of electrons, proportional to the mean density of matter over electron trajectory, exceeds at the altitude of 200 km the collision frequency at 350 km by more than 50 times. The diffusion velocities of electrons by altitude of reflection points are in the same ratio. The fundamental part of electrons (*) having reached the reflection altitude of 200 km, hit lower altitudes during a much lesser characteristic relaxation time of the initial distribution, where they lose their energy as they collide with the atmosphere matter. On that basis we assumed the distribution function of electrons with reflection points at h_{min} = 200 km, to be zero.

In the boundary condition (6) the quantity z_{max} corresponds to the altitude $\sim$ 3000 km. The correctness of the choice of z_{min} and z_{max} was corroborated by the calculation.

It was assumed that the density and the composition of the atmosphere do not vary with height for $h \geqslant 2000$ km. At these altitudes the atmosphere corresponds to electron density $n \sim 3 \cdot 10^3$ cm^{-3}. The geomagnetic field was approximated by a dipole field placed at the center of the Earth. In this case the integrals

$$I_1(z, R_e) = \oint \sum Z_i n_i L_i \frac{dl}{\sqrt{1 - z^2 b(l)}},$$

$$I_2(z, R_e) = \oint \sum (Z_i^2 + Z_i) L_i n_i \frac{\sqrt{1 - z^2 b(l)}}{b(l)} dl,$$

$$I_3(z R_e) = \oint \frac{dl}{\sqrt{1 - z^2 b(l)}}$$

in expressions (2) - (4) are written in the form

$$I_1(z, R_e) = 4R_e \int\limits_0^{\lambda_m} \sum Z_i L_i n_i \frac{\cos^4 \lambda \sqrt{4 - 3\cos^2 \lambda}}{(\cos^6 \lambda - z^2 \sqrt{4 - 3\cos^2 \lambda})^{1/2}} d\lambda, \tag{8}$$

$$I_2(z, R_e) = 4R_e \int\limits_0^{\lambda_m} \sum (Z_i^2 + Z_i) L_i n_i \cos^4 \lambda (\cos^6 \lambda - z^2 (\sqrt{4 - 3\cos^2 \lambda}))^{1/2} d\lambda, \tag{9}$$

$$I_3(z, R_e) = 4R_e \int\limits_0^{\lambda_m} \frac{\cos^4 \lambda \sqrt{4 - 3\cos^2 \lambda}}{(\cos^6 \lambda - z^2 \sqrt{4 - 3\cos^2 \lambda})^{1/2}} d\lambda,$$

$$z^2 = \frac{\cos^6 \lambda_m}{\sqrt{4 - 3\cos^2 \lambda_m}}, \tag{10}$$

where λ_m is the geomagnetic latitude of the reflection point of the electron with pitch-angle at the equator

Integrals (8) - (10) were calculated by computer. Plotted in Fig.1 are the dependences

$$F(z, R_e) = \frac{mv^2}{4\pi e^4} F(p, z, R_e) \quad \text{and} \quad K(z, R_e) = \frac{p^2 v}{4\pi e^4} K(p, z, R_e)$$

on the altitude h of the reflection point for four lines of force of the geomagnetic dipole. Functions $F(z, R_e)$ and $K(z, R_e)$, describing the deceleration and the scattering of electrons, depend on the penetration depth of the electron into the dense layers of the atmosphere (h) and the electron path in the geomagnetic field, that is, on the equatorial distance of the line of force R_e. Comparison of the values of $F(z, R_e)$ and $K(z, R_e)$, taken for the same value of h, but for different R_e, shows that to greater R_e correspond lower values of $F(z, R_e)$ and $K(z, R_e)$. This becomes understandable if we take into account that the trajectory of the electron, moving along the line of force with a great R_e, lies mainly in rarefied atmosphere, while the electron, moving along the line of force with small R_e, encounters in its path mostly dense layers of matter.

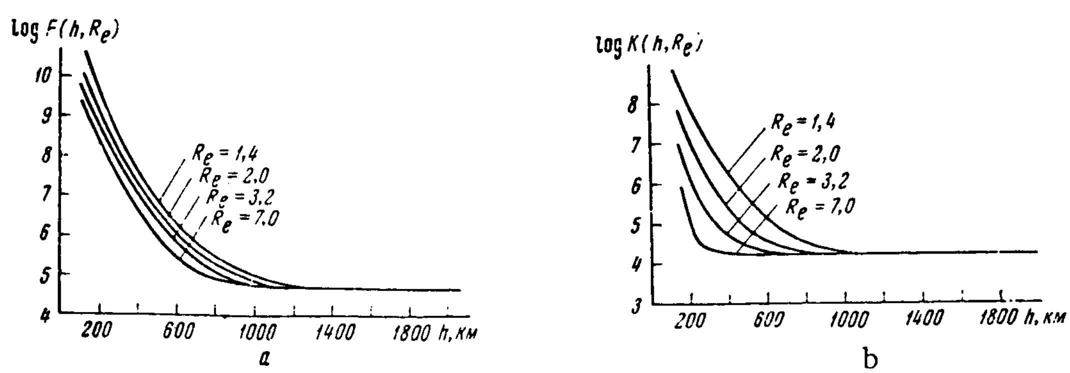

Fig.1

As a result of the calculation the time dependence of the distribution function of electrons with energy E = 1 Mev was found, with initial altitudes of reflection h_0 = 350, 500 and 750 km on the lines of force R_e = 1.4, 2,0, 3.2, 7, passing respectively in the intensity maximum of the inner belt, in the region of decreased intensity between belts, in the maximum of the outer belt and beyond the latter. The equatorial distances are measured in R_E. Assumed for the characteristic relaxation time was the time, upon the lapse of which 30 percent of the initial number of electrons with energy $E \geqslant 0.5E_0$ still remained. The thus determined relaxation time will be called in the following as electron lifetime τ in the radiation belt. The behavior of the distribution function was considered over the time interval from 0 to τ. Figure 2 illustrates the diffusion of electrons with energy 1 Mev by the altitude h of reflection points for three initial values of h_0 on the line of force R_e = 3.2. Inasmuch as the atmosphere density rises rapidly with decrease of altitude, while the lowering or the raising of the reflection point are are equally probable (Table 1), electrons diffuse rapidly into the dense layers of the atmosphere. In the latter the electrons perish rapidly as a result of deceleration.

TABLE 1

h	ρ, г·см$^{-3}$	N_2, см$^{-3}$	O_2, см$^{-3}$	O, см$^{-3}$	He, см$^{-3}$	H, см$^{-3}$	e, см$^{-3}$
150	$3 \cdot 10^{-12}$	$4,8 \cdot 10^{10}$	$7,5 \cdot 10^{9}$	$1,4 \cdot 10^{10}$	$1,1 \cdot 10^{7}$	$2,3 \cdot 10^{4}$	10^{5}
200	$3,1 \cdot 10^{-13}$	$4,2 \cdot 10^{9}$	$4,9 \cdot 10^{8}$	$3,1 \cdot 10^{9}$	$5,8 \cdot 10^{6}$	$1,6 \cdot 10^{4}$	10^{5}
250	$4,9 \cdot 10^{-13}$	$5,1 \cdot 10^{8}$	$4,5 \cdot 10^{7}$	$8,7 \cdot 10^{8}$	$3,7 \cdot 10^{6}$	$1,2 \cdot 10^{4}$	10^{5}
300	$1,9 \cdot 10^{-14}$	$1,5 \cdot 10^{8}$	$1,1 \cdot 10^{7}$	$4,2 \cdot 10^{8}$	$3,0 \cdot 10^{6}$	$1,1 \cdot 10^{4}$	10^{5}
400	$2,7 \cdot 10^{-15}$	$9,3 \cdot 10^{6}$	$4,7 \cdot 10^{5}$	$8,5 \cdot 10^{7}$	$2,0 \cdot 10^{6}$	$9,9 \cdot 10^{3}$	$5 \cdot 10^{5}$
500	$5,4 \cdot 10^{-16}$	$6,8 \cdot 10^{5}$	$2,4 \cdot 10^{4}$	$1,9 \cdot 10^{7}$	$1,3 \cdot 10^{6}$	$8,9 \cdot 10^{3}$	$3 \cdot 10^{5}$
600	$1,3 \cdot 10^{-16}$	$5,4 \cdot 10^{4}$	$1,3 \cdot 10^{3}$	$4,4 \cdot 10^{6}$	$9,2 \cdot 10^{5}$	$8,1 \cdot 10^{3}$	$5 \cdot 10^{4}$
700	$3,3 \cdot 10^{-17}$	$4,6 \cdot 10^{3}$	$9,9 \cdot 10^{1}$	$1,1 \cdot 10^{6}$	$6,5 \cdot 10^{5}$	$7,4 \cdot 10^{3}$	10^{4}
800	$1,3 \cdot 10^{-17}$	$6,8 \cdot 10^{2}$	$8,8 \cdot 10^{0}$	$3,6 \cdot 10^{5}$	$4,6 \cdot 10^{5}$	$6,8 \cdot 10^{3}$	$7 \cdot 10^{3}$
1000	$2,1 \cdot 10^{-18}$	$6,8 \cdot 10^{0}$	—	$2,6 \cdot 10^{4}$	$2,4 \cdot 10^{5}$	$5,8 \cdot 10^{3}$	$5 \cdot 10^{3}$
1200	$9,0 \cdot 10^{-19}$	—	—	$3,1 \cdot 10^{3}$	$1,3 \cdot 10^{5}$	$5,0 \cdot 10^{3}$	$5 \cdot 10^{3}$
1400	$4,8 \cdot 10^{-19}$	—	—	$2,9 \cdot 10^{2}$	$7,1 \cdot 10^{4}$	$4,3 \cdot 10^{3}$	$5 \cdot 10^{3}$
1600	$2,8 \cdot 10^{-19}$	—	—	$3,0 \cdot 10^{1}$	$4,0 \cdot 10^{4}$	$3,7 \cdot 10^{3}$	$5 \cdot 10^{3}$
1800	$1,6 \cdot 10^{-18}$	—	—	—	$2,4 \cdot 10^{4}$	$3,2 \cdot 10^{3}$	$5 \cdot 10^{3}$
2000	$1,0 \cdot 10^{-19}$	—	—	—	$1,4 \cdot 10^{4}$	$2,9 \cdot 10^{3}$	

Electrons with reflection points at great heights during a prolonged time pursue their motion in a rarefied atmosphere, where interactions with the medium are very scarce. The position of the distribution maximum of electrons on the line of force R_e = 3.2 as a function of time is shown in Fig.3. from which it may be seen that toward the moment of time τ the distribution maximum of electrons with h_0 = 350, 500 and 750 km shifts respectively to heights ∿ 500, 700 and 1200 km. As is shown by calculation, electrons with reflection heights < 1000 km diffuse into the dense layers of the atmosphere, hardly losing any energy. The energy distribution maximum shifts somewhat toward great energies on account of the fact that for low energy electrons the diffusion process unfolds more rapidly.

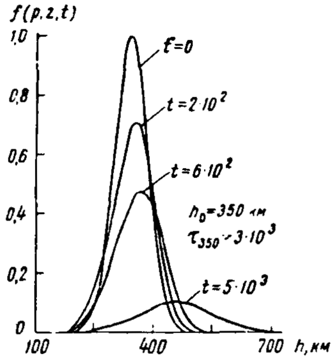

Fig.2,a

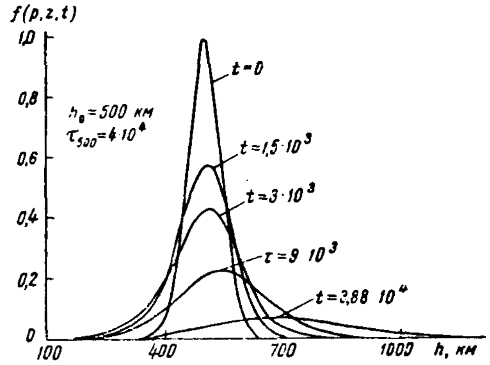

Fig.2,b

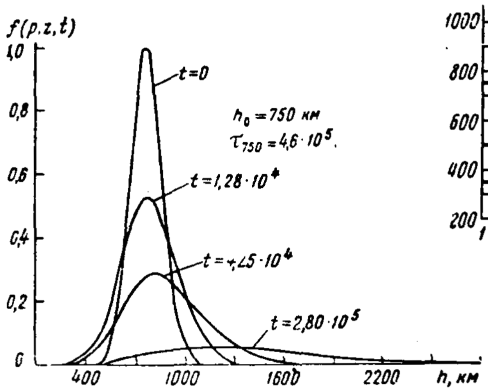

Fig.2, c

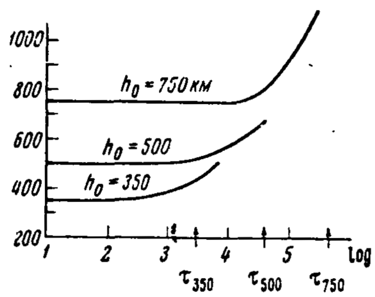

Fig.3

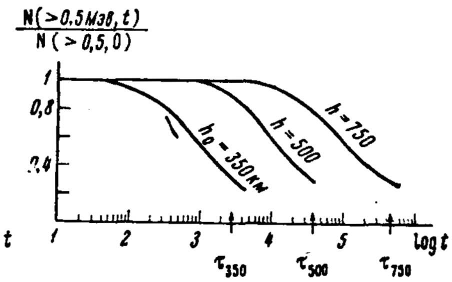

Fig.4

TABLE 2

R_e	h_0, км		
	350	500	750
1,4	$1,1\cdot10^3$	$1,8\cdot10^4$	$4,6\cdot10^5$
2	$1,2\cdot10^3$	$2,4\cdot10^4$	$4,6\cdot10^5$
3,2	$3,0\cdot10^3$	$4,0\cdot10^4$	$4,6\cdot10^5$
7,0	$3,2\cdot10^3$	$1,7\cdot10^4$	$8,5\cdot10^4$

The lifetimes of electrons with energy 1 Mev (τ, sec.) have been obtained in the work for the above enumerated initial heights of reflection points h_0 and lines of force R_e of the geomagnetic field. The temporal dependence of the ratio of the number of electrons at altitudes > 200 km to the number of electrons at the moment of time t = 0 for three different initial reflection heights on the line of force R_e = 3.2 is plotted in Fig.4. This ratio remains close to the unity so much the longer that the initial reflection point is higher. Such constance of the number of electrons situated on the line of force is explained by the fact that at the beginning of the relaxation process electrons lack the time to diffuse to lower altitudes $h \lesssim 200$ km (cf.Fig.2). According to the curves represented in Fig.4, and to similar curves obtained for other lines of force, we determined the lifetime of electrons. The lifetime dependence of electrons with energy 1 Mev on the equatorial distance of the line of force R for a fixed initial height of reflection points is plotted in Fig.5. The lifetimes of electrons with identical height of the reflection point coincide by order of magnitude for various lines of force of the geomagnetic field in the central dipole approximation. For reflection heights h_0 = 350, 500 and 750 km, the lifetimes of electrons are $\sim 10^3, 10^4, 10^5$ sec.

However, lifetime of electrons with identical height of the reflection point differ on various lines of force. As may be seen from Fig.5, for h_0 = 350, 500 km and equatorial distances R_e = 3.2, the lifetime of electrons is less for the lines of force with smaller R_e. Such time dependence of electrons' lifetime on R_e is explained by the fact that electrons, moving over a line of force with small equatorial distance, encounter over their path mainly a dense atmosphere. At h_0 = 750 km such a phenomenon is not observed, inasmuch as the density gradient is lesser at high altitudes, and moreover, for h ⩾ 2000 km, the atmosphere is considered as constant. Consequently, the mean density of matter over the trajectory of a particle with reflection point at altitude ∿700 km depends little on the equatorial distance of the line of force.

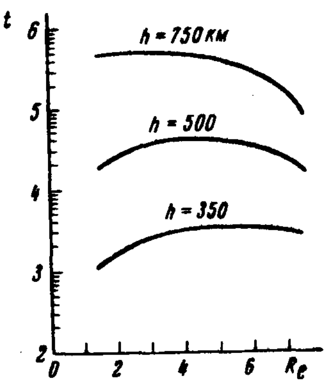

Fig.5

Calculation shows that for the assumed atmosphere model the lifetime of electrons with reflection points at h_0 = 750 km decreases with the transition to larger equatorial distances. This is explained by the fact that, by virtue of the character of the dipole field for one and the same variation of the pitch-angle in the scattering event, the variation of the reflection height is found to be greater for the lines of force with large equatorial distances. The lifetime τ dependence on R_e in case of great R_e may be estimated with the help of the kinetic equation, provided we neglect in it the term accounting for the deceleration. For an electron on a line of force with great R_e, at a height h ∿ 1000 km, the geomagnetic latitude of the reflection point is close to $\pi/2$. This is why

$$z^2 = \cos^6\lambda / \sqrt{4 - 3\cos^2\lambda} \sim \cos^6\lambda,$$

and the distance of the reflection point considered from the center of the Earth is

$$R = R_e \cos^2\lambda \sim R_e z^{2/3}.$$

Passing now to the variable $R = R_e z^{2/3}$, and without taking into account the deceleration, Eq,(1) may be written in the form

$$\frac{\partial f}{\partial t} = R_e^3 \frac{1}{R^2} \frac{\partial}{\partial R}\left(RK \frac{\partial f}{\partial R}\right).$$

The values of K for h ∿ 700 km differ little from one another in the considered atmosphere model. The dependence of the solution of the written equation on the equatorial distance R_e is taken into account by the multiplier R_e in the right-hand part of the equation. This is why for an identical reflection height h_0 ∿ 700 km

$$\frac{\tau_{R_e^{(1)}}}{\tau_{R_e^{(2)}}} \sim \left(\frac{R_e^{(2)}}{R_e^{(1)}}\right)^3.$$

For lines of force R_e = 3.2 and 7, and h_0 ∿ 700 km,

$$\tau_{3,2} / \tau_7 \sim 10.$$

The corresponding ratio of lifetimes obtained in the calculation at h_0 = 750 km constitutes ∿ 6. For lower reflection heights the estimate is found to be less precise, inasmuch as in this case the principal role is played by the difference in the various diffusion coefficients on different lines of force. It should be noted that the magnitude of the effect considered, obtained in the assumption of atmosphere homogeneity at heights > 2000 km, depends essentially on the altitude course cf matter density.

The lifetimes found as a result of numerical solution of Eq.(1), may be compared with the lifetimes obtained on the basis of the Christofilos theory of directed reflection point drift [1].

The lifetimes of electrons obtained with the help of the kinetic equation (1), in which scattering is considered as a diffusion process, agree by order of magnitude with the lifetimes computed according to the Christofilos theory for low altitudes h_0 = 350, 500 km. At greater heights (750 km) the difference increases notably.

Calculation shows that as time τ elapses, a notable share of initial reserve of particles continues to move along the line of force, being the object of reflection at heights notably exceeding the initial reflection height. Thus, on the line R_e = 3.2 the reflection point rises to a height of more than 100 km for ∿ 10 percent of electrons. The relaxation time of these electrons exceeds by more than 5 times the relaxation time of the initial distribution.

The calculation of the time dependence of the distribution function and of lifetime was performed for electrons with energy 1 Mev. However, if we take into account that in case of small reflection heights the deceleration is unsubstantial, we may drop in Eq.(1) the term taking into account the deceleration. On the basis of equation

$$\frac{\partial f}{\partial t} = \frac{1}{2z} \frac{\partial}{\partial z} \left\{ zK(p, z) \frac{\partial f}{\partial z} \right\}$$

and noting that $K(p, z) \sim 1/p^2 v$, we may estimate the lifetime of electrons of other energies. The lifetime τ_v of an electron, moving with a velocity v, is expressed by the lifetime τ_0 of an electron of 1 Mev energy by the relation

$$\tau_v \simeq \tau_0 \frac{v^3 \gamma^2}{v_0{}^3 \gamma_0{}^2},$$

where v_0 is the velocity of the electron with 1 Mev energy. For relativistic electrons $v \sim c$, and

$$\tau_E \simeq 0.45 \tau_0 E^2,$$

where E is the total energy of the electron in Mev.

We shall note in conclusion that the deceleration time of the electron exceeds by more than order the lifetime of electrons conditioned by Coulomb scattering.

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*** THE END ***

R E F E R E N C E S

1. N. C. CHRISTOFILOS. J. Geophys. Res., 64, 869, 1959.
2. W. M. MACDONALD, M. WALT. Ann. Phys., 15, 1, 44, 1961.
3. B. A. TVERSKOY. Geomagnetizm i Aeronomiya, 1, 6, 902, 1961.
4. M. WALT, W. M. MACDONALD. J. Geopnys. Res., 67, 5013, 1962.
5. W. M. MACDONALD, M. WALT. Ibid. 67, 5025, 1962.
6. G. A. TIMOFEYEV. Geomagnetizm i Aeronomiya. 5, 4, 754, 1965.
7. J. A. WELSH, R. L. KAUFMANN, W. H. HESS. J. Geophys. Res, 68, 685, 1963.
8. M. WALT. Ibid. 69, 3947, 1964.
9. L. D. LANDAU. ZhETF, 7, 203, 1937.
10. S. T. BELYAYEV. Fizika plazmy i problemy upravleniya termoyadernoy reaktsiyey (Physics of Plasma and Problems of Nuclear Recation Control). T.3. Izd-vo AN SSSR, 51, 1958.
11. V. M. PASKONOV, N. V. RYABIN'KINA. Chislennyye metody gazovoy dinamiki (Numerical Methods in Gas Dynamics). Sb.rabot VTS MGU vyp.6, MGU (Moscow State Univ.) 130, 1965.
12. T. HARRIS, W. PRIESTER. The theoretical models for the solar cycle variation of the upper atmosphere. NASA, (Prepr.) 1962.
13. K. L. BOWLES. Report to the XIV General Assembly of URSI, Tokyo, Sep.1963.

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1145 19th St.NW
WASHINGTON D. C. 20036.
Tel: 223-6700; 223-4930.

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